Title: Zipping on the Hypotenuse!

Brief Overview:

Students will apply their knowledge of the Pythagorean theorem to a real world situation. They will make a simulation of a potential piece of playground equipment called a "Zipper." Then they will test three different slopes of the Zipper's line to investigate the relationship of the slope of the line to the time it takes an object to slide down the line. They will choose the most appropriate measure of central tendency which will reflect their data. They will graph their data. Finally, they will write a letter to a company informing them of how much line they will need in order to complete the construction of the Zipper.

Links to NCTM Standards:

- **Mathematics as Problem Solving**
  Students will demonstrate their ability to solve problems in a cooperative atmosphere by working together to build a zip-line.

- **Mathematics as Communication**
  Students will demonstrate their ability to communicate mathematically by explaining how to use the Pythagorean theorem and scale drawings to determine the required length of zip-line.

- **Mathematics as Reasoning**
  Students will demonstrate their ability to reason mathematically by writing a letter to a company to ask for a specific length of line to make the zip-line.

- **Mathematical Connections**
  Students will demonstrate their ability to connect mathematics topics within the discipline by working with geometry, measurement, proportions, and statistics. They will connect mathematics with the scientific method. They will use mathematics to complete a writing prompt.

- **Computation and Estimation**
  Students will use proportions to calculate measurements for a scale drawing.

- **Measurement and Geometry**
  Students will measure the parts needed to build a model of a piece of playground equipment. Students will apply measurement to a real-world problem solving situation. Students will apply the Pythagorean theorem in order to find the missing side of a right triangle.

- **Statistics**
  Students will collect, organize, display, and interpret data about the speed of an object going down the zip-line. They will determine the best measure of central tendency for the data, and calculate it. They will use the data analysis in order to write a letter of persuasion to a company in order to receive the appropriate length of line.

Grade/Level:

Grade 7/8

Duration/Length:

This activity should take three days, including the assessment.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Using the Pythagorean theorem
- Applying proportions to scale drawings
- Measuring with the metric system
- Finding measure of central tendency
- Constructing and interpreting a scatter plot

Objectives:

Students will be able to:

- relate the slope of a line to the speed of an object.
- find the most appropriate measure of central tendency of the data they collect.
- apply the Pythagorean theorem.
- make a scale drawing.
- write a letter of persuasion.

Materials/Resources/Printed Materials: (for each group)

- 1 meter stick
- 1.2 meters of light gauge wire
- Calculator that has the square root function
- Stop watch
- Paper clip
- Object (i.e. fishing weight, heavy metal washer, etc.)
- Approximately 12' of masking tape
- Class set of metric rulers
- Class set of Congratulation/Simulation worksheets
- Class set of Data sheets
- Class set of Scale Drawing worksheets

Development/Procedures:

Day 1:

- Hand out the Congratulations/Simulation Worksheets. Have the class read the first few worksheet pages and answer any questions that they may have.

- Organize the class into groups of four. Have the students complete Question Set A, then proceed with the investigation.

- The work for Day 1 may extend into Day 2 depending upon time available, levels of students, or extenuating circumstances.

Days 2 & 3:

- Hand out the Scale Drawing Worksheet and a metric ruler to each student.
- Have the students complete the scale drawing of a zip-line, then answer the questions.
- Have the students write a letter to the Hyper-Krispies breakfast company.

Performance Assessment:

The students will be assessed on Day 2 as they complete the Scale Drawing worksheet individually. They will be assessed on Day 3 by writing a letter to a company.

Extension/Follow Up:

- The students may use a graphing calculator to graph their scatter plot, determine the measure of central tendency (if the mean), and apply the Pythagorean theorem.

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Congratulations! Your teacher has just explained that your classes' idea of designing and constructing a zip-line playground apparatus has been chosen as finalist in a contest sponsored by the *Hyper-Krispies* breakfast cereal company! It is now up to your class to finalize your design by building a model of your apparatus and using your knowledge of mathematics to complete a parts list. You will then need to write a letter of explanation to *Hyper-Krispies* describing your final design and the processes used to obtain it.

Zip-line Simulation

You are now going to investigate the effect of a zip-line's slope on the time that it takes for an object to reach the end of the line. To do this you will need to completely read these laboratory instructions and answer Question Set A before performing the experiment.

The question you will strive to answer is as follows:

Problem: Does the slope of a zip-line affect the time it takes an object to slide to the end of the line?

You will need to obtain these materials from your teacher at the proper time:

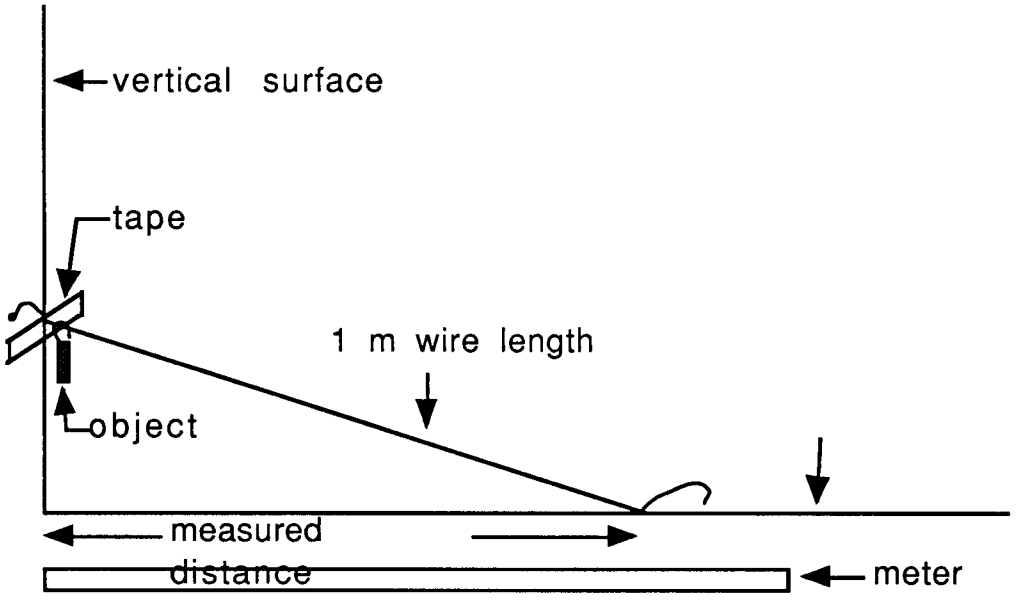
Materials: (for each group)

1 meter stick
1.2 meters of light gauge wire
Calculator
Stop watch
Paper clip
Object (i.e. fishing weight, heavy metal washer, etc.)
Approximately 12' of masking tape

This is the procedure that you will follow to perform your simulation:

Procedure:

1. Find a surface vertical to the floor (i.e. wall space, back of door) clear of any obstructions for a 1 meter distance.
2. On the vertical surface, tape one end of the wire 40 cm from the floor.
3. Stretch the wire to the floor so that the length along the wire is 1 meter. (There should be a short length of wire that can be used as a handle.)
4. Measure the distance along the floor from directly below the taped wire end to the point where the 1 meter length of wire touches the floor. Record this datum (see **Data**)
5. Attach the object to the paper clip and hook it onto the taped end of the wire next to the vertical surface.
6. Simultaneously start the stopwatch and release the object.
7. When the object hits the ground, stop the stopwatch. Record this datum. (see **Data**) Repeat this step twice more.
8. Move the taped end of the wire to a height of 60 cm on the vertical surface and repeat steps 2 through 7.
9. Move the taped end of the wire to a height of 80 cm on the vertical surface and repeat steps 2 through 7.



Day 1 Worksheets

Before you perform this experiment, you must state a prediction of the relationship between the **independent variable** and the **dependent variable.** The **independent variable** is the quantity that is controlled or changed by the experimenter. The **dependent variable** is the quantity that is measured by the experimenter and depends on how the independent variable is set.

Question Set A: (Answer in complete sentences.)

1. What is the independent variable for your experiment?

   _____
   _____
   _____
   _____.

2. What is the dependent variable for your experiment?

   _____
   _____
   _____
   _____.

3. State your prediction for the relationship between the independent variable and the dependent variable:

   As the **independent variable** increases/decreases (circle one),

   the **dependent variable** increases/decreases (circle one).

4. What type of correlation is your prediction?

   _____
   _____
   _____
   _____.

Data:

Zip-line Simulation Data Chart

	Initial Height cm	Measure distance from wall cm	Wire length cm	TIME TRIALS Trial 1 (sec)	Trial 2 (sec)	Trial 3 (sec)	Measure of central tendency (sec)
Event 1	40		100				
Event 2	60		100				
Event 3	80		100				

Data Analysis:

Question Set B: (Answer in complete sentences.)

1. Choose the measure of central tendency that you wish to use to represent the three trial times for each event. Explain the reasoning you used for your choice.

_____

_____

_____

_____

_____

2. Fill in the last column of the data chart using your choice for central tendency.
3. What geometric shape is formed by the floor, vertical surface, and zip-line?

_____

_____

4. Which side listed in question #3 represents the hypotenuse?

_____

_____

5. For each triangle, calculate the theoretical distance from the wall (using the Pythagorean theorem). Fill in the following table:

Height	**40**	**60**	**80**
Theoretical Distance			
Measured Distance			

Explain the possible reasons for any differences.

_____

_____

_____

_____

_____

_____

_____

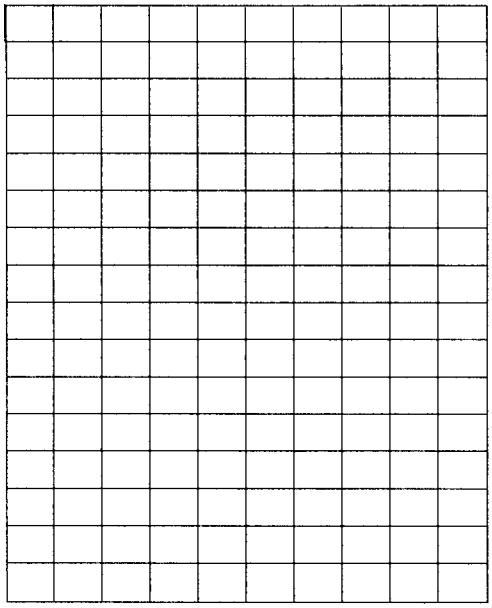
_____

_____

_____

Graphing:

To better visualize data patterns, graphs are often used. Graph the initial height vs. measure of central tendency for each event using a scatter plot.



Question Set C: (Answer in complete sentences)

1. Did your prediction in Question Set A #3 agree with your data?

   _____

2. Using your graph, make a prediction for the length of time it would take your object to slide down the 1 meter long wire if it were released from a 130 cm height. Explain how you arrived at your answer.

   _____
   _____
   _____
   _____
   _____

3. Which event best simulates a playground apparatus that you would choose for your school? Justify the reason for your choice.

   _____
   _____
   _____

Day 2 and Day 3 Scale Drawing Worksheets

We realize that we have trees on our property that would be ideal for the zip-line!

Listed below are the characteristics of the two trees:
-Twenty-one meters apart
-Twenty-five meters tall

Listed below are the guidelines for the apparatus:
-Attach the zip-line 8 meters high on one of the trees
-Attach the other end of the zip-line to the base of the second tree
-Draw a scale model on a separate sheet of paper of what the apparatus will look like
-The scale for the model is 1 centimeter = 2 meters

Answer the following questions in <u>complete sentences.</u> Use sound mathematical thinking.

1. Use a metric ruler to measure the length of the zip-line in your scale drawing to the nearest tenth of a centimeter. Use the given scale and set up a proportion to determine the actual length of the zip-line. Record your results.

_____

_____

_____

2. Now use the Pythagorean theorem ($a^2 + b^2 = c^2$) to determine the actual length of the zip-line and round to the nearest tenth. Record your results.

_____

_____

_____

3. Did the length of the zip-line determined from your scale drawing equal the length of the zip-line determined using the Pythagorean theorem? Explain the reason for any differences you notice.

4. You decide to increase the excitement by attaching the zip-line at a height of 10 meters. How much zip-line is required for this adjustment to the height? Explain how you calculate the amount of zip-line required.

5. You find out that you will not be able to purchase a new zip-line. If you attach the line you already have at a height of 10 meters, how far from the tree will you attach the zip-line to the ground? Explain in detail how you determine your answer.

Day 2 and Day 3 Scale Drawing Worksheets
ANSWER KEY

We realize that we have trees on our property that would be ideal for the zip-line!

Listed below are the characteristics of the two trees:
-Twenty-one meters apart
-Twenty-five meters tall

Listed below are the guidelines for the apparatus:
-Attach the zip-line 8 meters high on one of the trees
-Attach the other end of the zip-line to the base of the second tree
-Draw a scale model on a separate sheet of paper of what the apparatus will look like
-The scale for the model is 1 centimeter = 2 meters

Answer the following questions in <u>complete sentences.</u> Use sound mathematical thinking.

1. Use a metric ruler to measure the length of the zip-line in your scale drawing to the nearest tenth of a centimeter. Use the given scale and set up a proportion to determine the actual length of the zip-line. Record your results.

<u>Accept 11-11.4 cm for the length of the zip-line. (5 points)</u>

<u>The proportion is 1cm/2m=11.2cm/x.</u>

<u>Accept 22-22.8m for the actual length of the zip-line.</u>

2. Now use the Pythagorean theorem ($a^2 + b^2 = c^2$) to determine the actual length of the zip-line and round to the nearest tenth. Record your results.

<u>Accept 22.5 m for the length of the zip-line.(5 points)</u>

3. Did the length of the zip-line determined from your scale drawing equal the length of the zip-line determined using the Pythagorean theorem? Explain the reason for any differences you notice.

Answers will vary in determining whether measurements are equal.

Explanation should include a reference to errors in drawing and

measuring. *(5 points)*

4. You decide to increase the excitement by attaching the zip-line at a height of 10 meters. How much zip-line is required for this adjustment to the height? Explain how you calculate the amount of zip-line required.

Twenty-three and three tenths meters of zip-line is needed. (5 points)

The answer should be calculated using the Pythagorean theorem, but

accept the scale drawing for partial credit.

5. You find out that you will not be able to purchase a new zip-line. If you attach the line you already have at a height of 10 meters, how far from the tree will you attach the zip-line to the ground? Explain in detail how you determine your answer.

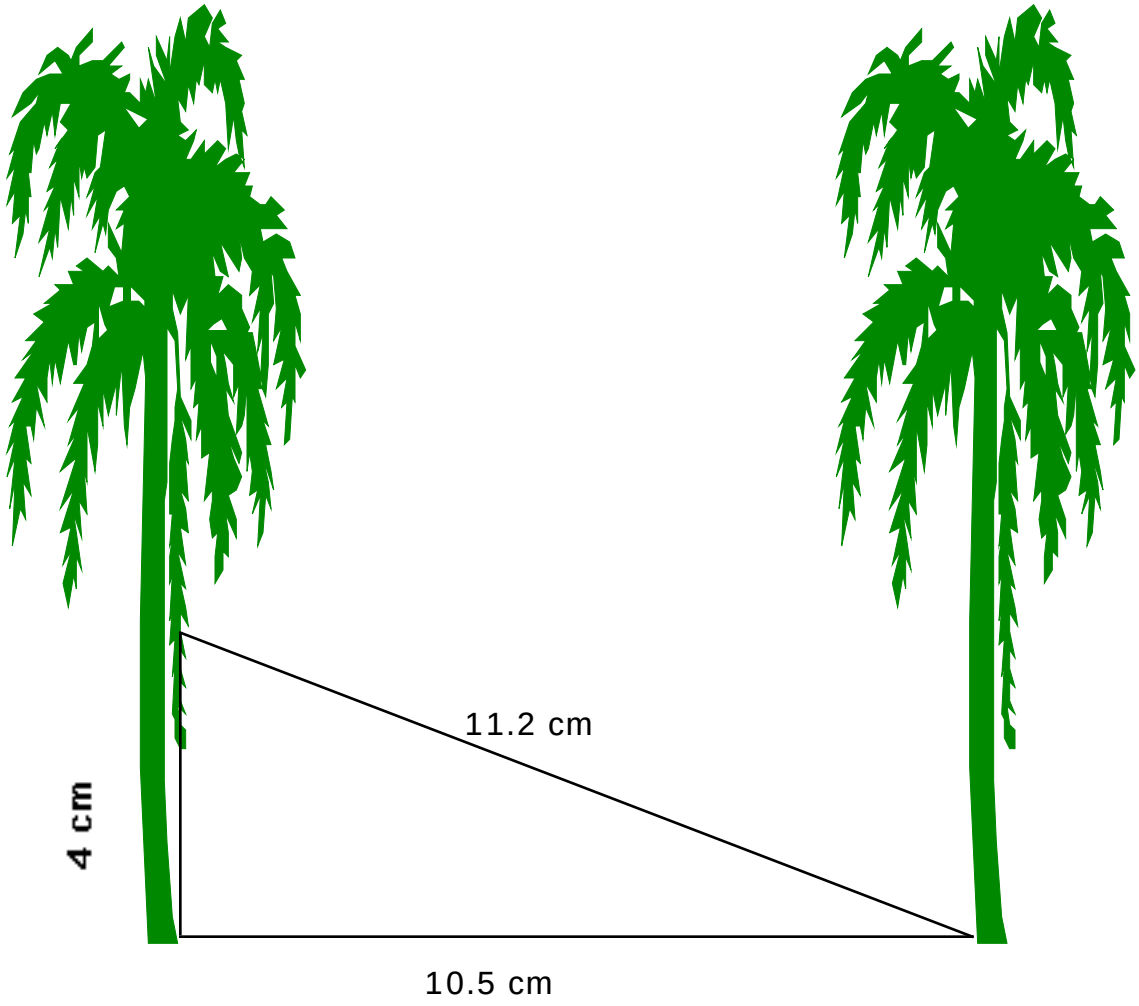
The zip-line will be attached 20.2 m from the base of the tree. (5 points)

The explanation should include reference to using the Pythagorean

theorem and the length of the original zip-line as their hypotenuse or

value for c.

Scale Drawing Answer Key



Scoring Rubric for Scale Drawing

3	All lengths exact and labeled properly. The triangle positioned properly with the right angle at the base of the tree.
2	Lengths within plus or minus 0.5 cm. The triangle positioned properly with the right angle at the base of the tree.
1	Lengths within plus or minus 1.0 cm. The triangle positioned properly with the right angle at the base of the tree.
0	All other answers.

Culminating Activity
Writing Prompt

Now that you have figured out how to make the zip-line, you must write a letter to the *Hyper-Krispies* breakfast cereal company. In the letter you must describe the final design of your zip-line, and the processes you used to make it. You must also include a sketch of the design.

Before you start to write, think about:

what length the zip-line will be
how high up the tree it will be attached
how far apart the trees will be
what mathematical procedures you use to find these measures
what mathematical terms you will need to use

Now write a letter to the *Hyper-Krispies* breakfast cereal company.

Scoring Rubric

3 Letter form proper, all think-abouts covered.

2 Letter form almost proper, most think-abouts covered.

1 Letter form almost proper, a couple think-abouts covered.

0 All other answers.